CALCULATION OF THE ABSORPTION OF γ-RAY ENERGY

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Presented is a Monte Carlo calculation of the distributions of absorbed energy and photon and energy fluxes in a hemisphere of water with a 1.26 MeV γ-ray point source at the center. Finite and infinite radii of the hemisphere are considered, and the results are compared with those obtained by numerical integration and by an approximate calculation. It is shown that in no case do the differences exceed 13%.

The design and operation of radiation apparatus involves a series of serious engineering and physical problems related to the study of the passage of radiations through matter. In contrast to problems of shielding physics, relatively thin layers of matter, usually not in excess of 30-50 cm in the case of a water-equivalent medium, are concerned. At the same time, the error in the results, especially in the analysis of the distribution of absorbed energy, must not be worse than 10%. In this connection, the extensive information obtained in the solution of problems of shielding physics is not always applicable.

The method used in this paper consists in constructing a probability model of the process of the passage of γ-radiation through matter and in tracking a large number of trajectories (histories) of different independent γ-quanta, each of which behaves according to the probability model constructed. Each trajectory is a single experiment, and the accuracy of the method depends upon the total number of such experiments [1-3].

The calculation was performed on the two-address Minsk-1 electronic computer. Previous calculations were made on computers superior to the Minsk-1 in possessing a considerably greater storage capacity and floating point arithmetic [4, 5], in connection with which a special program was developed. Since the Minsk-1 is a machine with fixed point arithmetic, a floating point was introduced in the program. With the aim of reducing program length, pseudo-instructions were used to replace arithmetic operations. An interpretive standard subroutine serves for deciphering these pseudo-instructions and executing the corresponding arithmetic operations in the floating point regime.

The following parameters are printed out: rectangular and polar coordinates of points at which collisions occur; energy losses in each collision and the energy with which a γ-quantum arrives at the collision point; type of interaction. If the γ-quantum emerges backwards, the escape geometry is also registered. The history is completed with photoabsorption or escape of a quantum from the region considered. The history is also interrupted if a quantum reaches a cutoff energy α_{min} for which the probability of photoabsorption considerably exceeds the probability of Compton scattering. This energy is taken to be 0.04 MeV, and it is assumed that at this energy the quantum undergoes photoabsorption. Analysis of the data obtained shows that in this case the resulting error does not exceed 1%. Only Compton scattering and the photoeffect were considered in the calculations. 1017 histories were evaluated. A block diagram of the program is shown in Fig. 1. Let us consider the functions of the operators: χ defines the azimuthal scattering angle; L computes the range of the γ-quantum in the material X, Y, Z, r compute the coordinates of the point of interaction of the quantum with an electron of the material; comparison of r and Z keeps track of the quantum until it passes through the
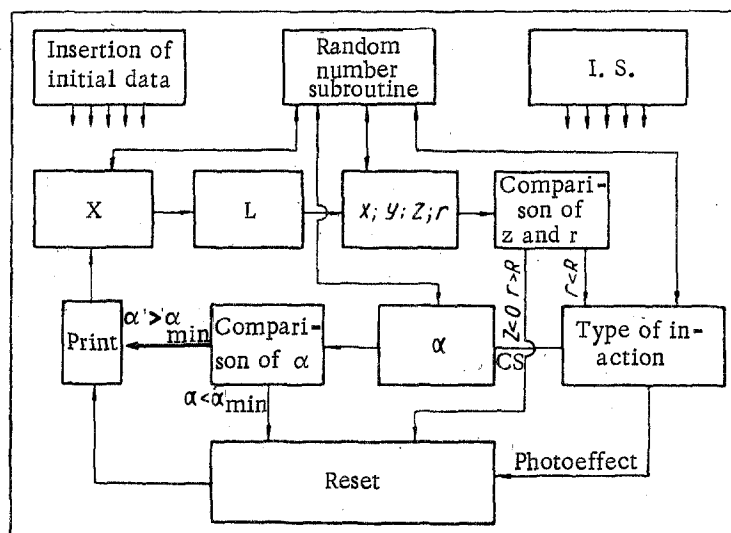

Fig. 1. Block diagram of the program.

layer (r > R) or escapes backwards (Z < 0); the "type of interaction" operator denotes the interaction process (if μ_{qb}/μ < A, Compton scattering occurs; if μ_{qb}/μ > A, photoabsorption); α' determines the energy of the quantum after scattering (calculated by the method of successive approximations, the first approximation being taken as $\alpha' = \alpha$); comparison of α verifies fulfillment of the condition $\alpha' > \alpha_{min}$; the "reset" operator is needed for preparing the program to calculate a new history, i.e., for restoring the original form of the variable instructions.

To obtain pseudo-random numbers a special program based on choosing the middle of the product of two numbers [3] was compiled. The process consists in multiplying together two n-digit random numbers A_k and A_{k+1} (using multiplication with double precision); a 2n-digit product is obtained. The middle n-digits of the product are taken as A_{k+2}. Then the process is repeated with A_{k+1} and A_{k+2} to obtain the succeeding number, and so on. The program provides for a check on the uniformity of distribution of the sequence of pseudo-random numbers over the interval (0, 1).
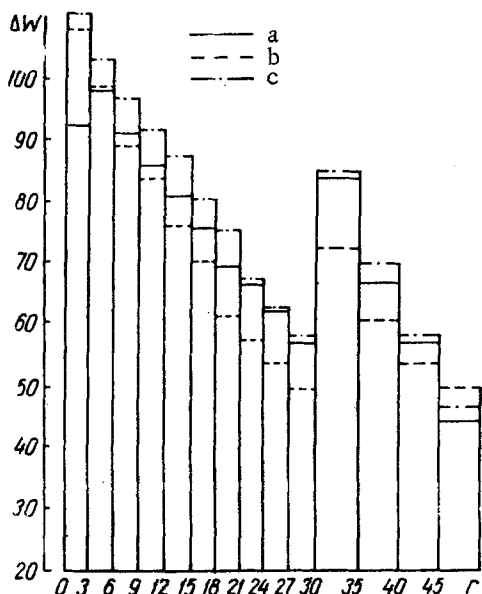

Fig. 2. Energy absorbed in hemispherical annular layers (MeV). 1017 γ-quantum histories were calculated: a) by Monte Carlo, b) by the gamma-method, c) by numerical integration.

To calculate the energy absorbed, the irradiated medium is divided into concentric hemispheres at radial intervals of 3 and 5 cm. For each collision, the difference in energy of the incoming and outgoing γ-quanta was recorded. The energy losses in collisions were then summed over each spherical layer. In view of the small range of secondary electrons, it was assumed that all the energy losses were absorbed directly at the point of interaction. The distribution of absorbed energy in an infinite hemisphere thus obtained is shown in the histogram in Fig. 2, which also shows the corresponding values obtained by the gamma-method. According to the latter, the absorbed dose from a point source may be obtained from W $\approx$ SE$_0$ $\times$ $\times$ exp$(-\gamma r) \times \gamma/4\pi r^2$. Then, for the energy absorbed in the layer between R$_1$ and R$_2$, we get:

$$\Delta W = \int_{R_1}^{R_2} W 4\pi\, r^2 dr =$$

$$= SE_0 [\exp(-\gamma R_1) - \exp(-\gamma R_2)].$$

For comparison, the same histogram shows values of ΔW obtained by numerical integration using values of the absorbed energy build-up factor obtained by integrating the data of [6]. Clearly, the differences between values of the absorbed energy obtained by the Monte Carlo method and by numerical integration do not exceed 8%. For small thicknesses, this is linked with the fact that with the assumed geometry contributions to the absorbed energy due to quanta reflected from the back half-space are excluded. The appearance of a maximum in the absorbed energy at a depth of 3-6 cm has the same explanation. A study of the effect of the finiteness of the medium of the distribution of absorbed energy in a hemisphere qualitatively confirms the relations obtained by Spencer and Berger [7]. However, our values for the corrections for a finite medium are 0.6-0.9 times those given in [7]. Comparison of the values of the absorbed energy obtained by the Monte Carlo and gamma-methods permits certain conclusions on the applicability of the gamma-method to the calculation of the absorbed dose. For the thicknesses shown in Fig. 2, the method gives results that are somewhat too low (within 13%). The anomalous behavior of the method at the boundary of the irradiated object (see histogram) is related to the influence of edge effects on the Monte Carlo calculation.

In calculating the energy and γ-quanta fluxes, we recorded each crossing by a γ-quantum of concentric spherical surfaces with the same spacing as used in the absorbed energy calculations, with particular attention to quanta scattered from hemispheres of greater radius to those of lesser radius. Values of the energy and number of quanta build-up factors are shown in Fig. 3. The difference between the energy build-up factors obtained by the Monte Carlo method and in [6] does not exceed 10%. At small thicknesses, as in the case of the absorbed dose build-up factor, the effect of the finiteness of the medium is not great. However, in the case of the number build-up factor the effect is more considerable. With increasing thickness, this factor increases rapidly, due to the decrease in energy of the scattered quanta. The mean energy of the scattered quanta as a function of the depth of penetration for a hemisphere of radius 50 cm has the following values; for r = 3 cm, E = 0.63 MeV; 9—0.56; 15—0.49; 18—0.46; 21—0.43; 24—0.41; 27—0.38; 30—0.36; 35—0.32; 40—0.29; 50—0.25.
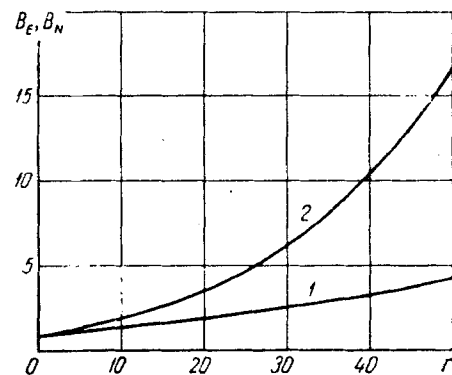

Fig. 3. 1) Energy build-up factor, 2) build-up factor for number of γ-quanta.

There follows a detailed formulation of each operator used in calculating the quantum trajectories in water by the Monte Carlo method:

$$\chi = 2\pi A; \quad L = \frac{1}{\mu(\alpha)} \ln \frac{1}{1-A} ;$$

$$\mu(\alpha) = \mu_\kappa + \mu_\phi; \quad \mu_\kappa = 0.082388 \frac{\alpha^2 - 2\alpha - 2}{\alpha^3} \ln(1 + 2\alpha) +$$

$$+ 2 \frac{2 + 8\alpha + 9\alpha^2 + \alpha^3}{\alpha^2(1 + 2\alpha)^2} ; \quad \mu_\phi^* = 0.1352537 \, 10^{-4}(\alpha)^{-3.252434} + 0.48560688 \, 10^{-4};$$

$$X_{n+1} = X_n + L_{n+1} \sin\theta_{n+1} \cos\varphi_{n+1}; \quad Y_{n+1} = Y_n + L_{n+1} \sin\theta_{n+1} \sin\varphi_{n+1};$$

$$Z_{n+1} = Z_n + L_{n+1} \cos\theta_{n+1};$$

$$\cos\theta_{n+1} = \cos\omega_{n+1}\cos\theta_n + \sin\omega_{n+1}\sin\theta_n\cos\chi_{n+1}; \quad \cos\omega_{n+1} = \frac{\alpha' + \alpha'\alpha - \alpha}{\alpha'\alpha} ;$$

$$\sin(\varphi_{n+1} - \varphi_n) = \frac{\sin\chi_{n+1}\sin\omega_{n+1}}{\sin\theta_{n+1}} ;$$

$$\cos(\varphi_{n+1} - \varphi_n) = \frac{\cos\omega_{n+1} - \cos\theta_n\cos\theta_{n+1}}{\sin\theta_n\sin\theta_{n+1}} ;$$

$$r = \sqrt{X^2 + Y^2 + Z^2}.$$

The quantity α' is found from

$$A = \left[\frac{\alpha + 4}{2\alpha^2} + \frac{1}{\alpha'\alpha^2} - \frac{2\alpha + 1}{\alpha^4}\alpha' - \frac{(\alpha')^2}{2\alpha^3} + \frac{\alpha^2 - 2\alpha - 2}{\alpha^3}\ln\frac{\alpha}{\alpha'} \right] \times$$

$$\times \left[\frac{\alpha^2 - 2\alpha - 2}{\alpha^3}\ln(2\alpha + 1) + 2\frac{2 + 8\alpha + 9\alpha^2 + \alpha^3}{\alpha^2(2\alpha + 1)^2} \right]^{-1}$$

NOTATION

E_0 — initial energy of γ-quanta; α and α' — energies of quanta before and after scattering ($\alpha = E/0.51$); A — random number; $\mu(\alpha)$ — coefficient of attenuation of γ-radiation; μ_ϕ — coefficient of photoabsorption; γ — true absorption coefficient; X, Y, Z — rectangular coordinates of point of interaction; r — distance from source to point of interaction; L — range of quanta; R — radius of hemisphere; S — intensity of radiation source; W — energy absorbed at point of interaction; ΔW — energy absorbed in annular hemispherical layer; χ — azimuthal scattering angle; μ_K — coefficient of Compton scattering; the indices n and n + 1 relate to two successive collisions; θ_n — angle between z axis and direction of motion of a quantum after the n-th scattering; φ_n — angle between the plane XOZ and the plane passing through the velocity vector of the quantum after the n-th collision and the Z axis; ω — scattering angle of quantum in Compton scattering.

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*The equation for μ_ϕ was suggested by V. A. El'tekov and B. M. Terent'ev.